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Chaplygin gas cosmology—unification of dark matter and dark energy

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Abstract

The models that unify dark matter and dark energy based upon the Chaplygin gas fail owing to the suppression of structure formation by the adiabatic speed of sound. Including string theory effects, in particular the Kalb–Ramond field, we show how nonadiabatic perturbations allow a successful structure formation.

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In contrast to the standard assumption that dark matter and dark energy are distinct, there stands the hypothesis that both are different manifestations of a single entity. The first definite model of this type [1–3] is based on the Chaplygin gas, an exotic fluid with an equation of state

$$p = -A/\rho. \quad (1)$$

Subsequently, the generalization to $p = -A/\rho^\alpha$, $0 \leq \alpha \leq 1$, was given [4] and the term ‘quartessence’ coined [5] to describe unified dark matter/energy models.

One of the most appealing aspects of the original Chaplygin gas model is that it is equivalent to the Dirac–Born–Infeld (DBI) description of a Nambu–Goto membrane [6, 7]. However, string theory D-branes possess three features that are absent in the simple Nambu–Goto membrane action: (i) they support an Abelian gauge field A_μ reflecting open strings with their ends stuck on the brane, (ii) they couple to the dilaton, and (iii) they couple to the (pull back of) Kalb–Ramond [8] antisymmetric tensor field $B_{\mu\nu}$ which, like the gravitational field $g_{\mu\nu}$, belongs to the closed string sector. Consider a p -dimensional D-brane with coordinates x^μ , $\mu = 0, 1 \dots p$, moving in the $(p + 1)$ -dimensional bulk with coordinates X^a , $a = 0, 1 \dots p + 1$. In the string frame the action is given by [9]

$$S_{\text{DBI}} = -\sqrt{A} \int d^{p+1}x e^{-\Phi} \sqrt{(-1)^p \det(g^{(\text{ind})} + B)}, \quad (2)$$

where $g_{\mu\nu}^{(\text{ind})}$ is the induced metric or the ‘pull back’ of the bulk spacetime metric G_{ab} to the brane

$$g_{\mu\nu}^{(\text{ind})} = G_{ab} \frac{\partial X^a}{\partial x^\mu} \frac{\partial X^b}{\partial x^\nu}. \quad (3)$$

The field \mathcal{B} is an antisymmetric tensor field that combines the Kalb–Ramond and the electromagnetic fields $\mathcal{B}_{\mu\nu} = B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}$. Let us choose the coordinates such that $X^\mu = x^\mu$, and let the $(p+1)$ th coordinate $X^{p+1} \equiv \theta$ be normal to the brane. From now on, we set $p = 3$ and consider a 3-brane universe in a $(4+1)$ -dimensional bulk. Then

$$G_{\mu\nu} = g_{\mu\nu}, \quad \mu = 0, \dots, 3; \quad G_{\mu 4} = 0; \quad G_{44} = -1. \quad (4)$$

After a few algebraic manipulations similar to [10], the DBI action may be written as

$$S_{\text{DBI}} = \int d^4x \sqrt{-\det g} \mathcal{L}_{\text{DBI}}, \quad (5)$$

$$\mathcal{L}_{\text{DBI}} = -\sqrt{A} e^{-\Phi} \sqrt{(1-\theta^2)(1+\mathcal{B}^2) - \theta\mathcal{B}^2\theta - (\mathcal{B} \star \mathcal{B})^2}, \quad (6)$$

where we have used the abbreviations

$$\begin{aligned} \theta^2 &= g^{\mu\nu} \theta_{,\mu} \theta_{,\nu}, & \mathcal{B} \star \mathcal{B} &= \frac{1}{8\sqrt{-\det g}} \epsilon^{\mu\nu\rho\sigma} \mathcal{B}_{\mu\nu} \mathcal{B}_{\rho\sigma}, \\ \mathcal{B}^2 &= \frac{1}{2} \mathcal{B}_{\mu\nu} \mathcal{B}^{\mu\nu}, & \theta\mathcal{B}^2\theta &= \theta_{,\mu} \mathcal{B}^\mu{}_\nu \mathcal{B}^{\nu\rho} \theta_{,\rho}. \end{aligned}$$

Neglect for the moment the dilaton and the \mathcal{B} field. The DBI action (6) then reduces to a scalar Born–Infeld action $S_{\text{BI}} = \int d^4x \sqrt{-\det \bar{g}} \mathcal{L}_{\text{BI}}$ with the Lagrangian

$$\mathcal{L}_{\text{BI}} = -\sqrt{A} \sqrt{1-\theta^2}. \quad (7)$$

It may be easily shown [2] that this Lagrangian yields the equation of state (1).

In a homogeneous model the conservation equation $T^{\mu\nu}{}_{;\nu} = 0$ yields the density as a function of the scale factor $\rho(a) = \sqrt{A + B/a^6}$, where B is an integration constant. The Chaplygin gas thus interpolates between dust ($\rho \sim a^{-3}$) at large redshifts and a cosmological constant ($\rho \sim \sqrt{A}$) today, and hence yields a correct homogeneous cosmology.

The inhomogeneous Chaplygin gas based on a Zel’dovich-type approximation has been proposed [2], and the picture has emerged that on caustics, where the density is high, the fluid behaves as cold dark matter, whereas in voids, $w = p/\rho$ is driven to the lower bound -1 producing acceleration as dark energy. Soon, however, it has been shown that the naive Chaplygin gas model does not reproduce the mass power spectrum [11] and the CMB [12, 13].

The physical reason is that although the adiabatic speed of sound, defined by $c_s^2 = (\partial p/\partial \rho)_S$ is small until $a \sim 1$, the accumulated comoving acoustic horizon

$$d_s = \int dt c_s/a \simeq H_0^{-1} a^{7/2} \quad (8)$$

reaches Mpc scales by redshifts of twenty, frustrating the structure formation even into a mildly nonlinear regime [14]. In the absence of caustic formation, the density contrast described by a linearized evolution equation

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta - \frac{c_s^2}{a^2}\Delta\delta = 0 \quad (9)$$

undergoes damped oscillations that are in gross conflict with observations. The root of the structure formation problem is the last term on the left-hand side of (9). The solution in k -space $\delta_k = a^{-1/4} J_{5/4}(d_s k)$ shows the asymptotic behaviour:

$$\delta_k \sim a \quad \text{for } d_s k \ll 1; \quad \delta_k \sim \frac{\cos d_s k}{a^2} \quad \text{for } d_s k \gg 1. \quad (10)$$

Hence, the perturbations undergo damped oscillations at the scales below d_s .

One simple way to save the Chaplygin gas is to suppose that nonadiabatic perturbations cause the pressure perturbation δp to vanish [15], and with it the acoustic horizon. To achieve this, it is necessary to add new degrees of freedom [16] which, to some extent, spoil the simplicity of quintessence unification.

To illustrate this, consider the full DBI action (5) with (6) which contains extra degrees of freedom in terms of the dilaton Φ and the tensor field \mathcal{B} . It is important to stress that all three fields, the dilaton, the scalar DBI field θ , and the Kalb–Ramond field originate from an ultimate theory in the context of string/M theory and, unlike in quintessence models, each field affects both dark matter and dark energy. The full action

$$S = \int d^4x \sqrt{-\det g} (\mathcal{L}_b + \mathcal{L}_{\text{DBI}}) \quad (11)$$

contains, in addition to the DBI action, the bulk terms

$$\mathcal{L}_b = \frac{1}{2\kappa^2} e^{-2\Phi} \left(-R - 4g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} + \frac{1}{12} \mathcal{B}_{[\mu\nu,\sigma]} \mathcal{B}^{[\mu\nu,\sigma]} \right), \quad (12)$$

where $\kappa^2 = 8\pi G$. It is convenient to write everything in Einstein's frame using the transformation $g_{\mu\nu} \rightarrow e^{2\Phi} g_{\mu\nu}$ and simultaneously rescaling $\theta_{,\mu} \rightarrow e^\Phi \theta_{,\mu}$ and $\mathcal{B}_{\mu\nu} \rightarrow e^{2\Phi} \mathcal{B}_{\mu\nu}$. In this way we obtain

$$\mathcal{L}_{\text{DBI}} = -\sqrt{A} e^{3\Phi} \sqrt{(1 - \theta^2)(1 + \mathcal{B}^2) - \theta \mathcal{B}^2 \theta - (\mathcal{B} \star \mathcal{B})^2}, \quad (13)$$

$$\mathcal{L}_b = \frac{1}{2\kappa^2} \left(-R + 2g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} + \frac{1}{12} \mathcal{B}_{[\mu\nu,\sigma]} \mathcal{B}^{[\mu\nu,\sigma]} + V(\Phi, \mathcal{B}) \right), \quad (14)$$

$$V(\Phi, \mathcal{B}) = \frac{1}{3} \Phi_{[\mu,\nu} \mathcal{B}_{\nu\sigma]} \mathcal{B}^{[\mu\nu,\sigma]} + \frac{1}{3} \Phi_{[\mu,\nu} \mathcal{B}_{\nu\sigma]} \Phi^{[\mu,\nu} \mathcal{B}^{\nu\sigma]}. \quad (15)$$

Applying the variational principle to \mathcal{L} , one may easily derive the energy-momentum tensor

$$T_{\mu\nu} = T_{\mu\nu}^b + T_{\mu\nu}^{\text{DBI}} \quad (16)$$

and the equations of motion for θ , Φ , and $\mathcal{B}_{\mu\nu}$. Neither $T_{\mu\nu}^b$ nor $T_{\mu\nu}^{\text{DBI}}$ is in the form of a perfect fluid but we may still define the corresponding ρ and p using the decomposition

$$T_{\mu\nu} = \rho u_\mu u_\nu - p h_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu + \pi_{\mu\nu}, \quad (17)$$

$$\rho = T_{\mu\nu} u^\mu u^\nu, \quad p = -\frac{1}{3} T_{\mu\nu} h^{\mu\nu}, \quad (18)$$

$$q_\mu = T_{\nu\rho} u^\nu h_\mu^\rho, \quad \pi_{\mu\nu} = T_{\rho\sigma} h_\mu^\rho h_\nu^\sigma + p h_{\mu\nu}. \quad (19)$$

Here, q_μ and $\pi_{\mu\nu}$ are the energy flux and the anisotropic pressure, respectively, and $h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ is a projection tensor.

Next, let us see the implications for linear perturbation theory. To study perturbations, it is convenient to work in the so-called temporal or synchronous gauge

$$ds^2 = dt^2 - a^2(\delta_{ij} - f_{ij}) dx^i dx^j, \quad (20)$$

where f_{ij} is treated as first-order departure from homogeneity. In addition to that, the density contrast $\delta = (\rho - \bar{\rho})/\bar{\rho}$, and the field perturbations $\delta_\theta = \theta - \bar{\theta}$ and $\delta_\Phi = \Phi - \bar{\Phi}$ are first-order perturbations about the homogeneous and isotropic background. The homogeneity and isotropy imply $\bar{\theta}_{,i} = 0$, $\bar{\Phi}_{,i} = 0$, $\bar{\mathcal{B}}_{\mu\nu} = 0$. As a consequence of this, the spatial derivatives $\Phi_{,i} = \delta\Phi_{,i}$ and $\theta_{,i} = \delta\theta_{,i}$, as well as the second order in $\mathcal{B}_{\mu\nu}$ count as first-order perturbations.

With these assumptions we may neglect higher order terms in the expression for the energy momentum and we find simple expressions for the density and the pressure associated with the DBI part $T_{\mu\nu}^{\text{DBI}}$ of the energy momentum:

$$\rho = T_{\mu\nu}^{\text{DBI}} u^\mu u^\nu = \sqrt{A} e^{3\Phi} \sqrt{\frac{1 + \mathcal{B}^2}{1 - \theta^2}},$$

$$p = -\frac{1}{3} T_{\mu\nu}^{\text{DBI}} h^{\mu\nu} = -\frac{A e^{6\Phi}}{\rho} \left(1 + \frac{1}{3} \mathcal{B}^2\right).$$

Then, the pressure perturbation is given by

$$\delta p = -\bar{p} \left(\frac{\delta \rho}{\bar{\rho}} + 6\delta\Phi - \frac{1}{3} \mathcal{B}^2 \right), \quad (21)$$

and the nonadiabatic cancellation scenario is realized by

$$\frac{\delta \rho}{\bar{\rho}} + 6\delta\Phi - \frac{1}{3} \mathcal{B}^2 = 0 \quad (22)$$

as an initial condition outside the causal horizon $d_c = \int dt/a \simeq H_0^{-1} a^{1/2}$. However, it is essential that once the perturbations enter the causal horizon d_c , at least one of the two components, $\delta\Phi$ or \mathcal{B}^2 , grows in the same way as the density contrast.

In the following we demonstrate that whereas the dilaton does not yield the desired cancellation of nonadiabatic perturbations, the Kalb–Ramond field might provide such a mechanism.

The dilaton

Retaining only the dominant terms, the dilaton perturbation $\delta\Phi$ satisfies [17]

$$\delta\ddot{\Phi} + 3H\delta\dot{\Phi} - \frac{1}{a^2} \Delta\delta\Phi \simeq 0, \quad (23)$$

with the solution in k -space

$$\delta\Phi_k = a^{-3/4} J_{3/2}(kd_c). \quad (24)$$

Then, once the perturbations enter the causal horizon d_c (but are still outside the acoustic horizon d_s), $\delta\Phi$ undergoes rapid damped oscillations, so that nonadiabatic perturbation associated with Φ is destroyed. This means that the nonadiabatic perturbations are not automatically preserved except at long, i.e., superhorizon, wavelengths where the naive Chaplygin gas has no problem anyway.

The Kalb–Ramond field

To estimate the effect of the \mathcal{B} field, we adopt the temporal gauge ansatz [16, 18]

$$\mathcal{B}_{0i} = 0, \quad \mathcal{B}_{ij} = \epsilon_{ijk} B^k. \quad (25)$$

Then the \mathcal{B} -field term in (21) is

$$\mathcal{B}^2 = \frac{B^i B^i}{a^4}. \quad (26)$$

Retaining the dominant terms, the field equation for B^i takes the form

$$\frac{d}{dt} \left(\frac{\dot{B}^i}{a} \right) - \frac{B_{,ji}^j}{a^3} + \frac{2\kappa^2 A}{\bar{\rho}} \frac{B^i}{a} = 0. \quad (27)$$

If we make the decomposition $B^i = B_\perp^i + B_\parallel^i$ with the transverse part satisfying $\partial_i B_\perp^i = 0$, the key point becomes evident: whereas the longitudinal part B_\parallel^i suffers the same problem as $\delta\Phi$ in (23), the transverse part does not experience spatial gradients.

As $H^2 = (\dot{a}/a)^2 = \kappa^2 \bar{\rho}/3$, the last term in (27) is of the order $H^2 A/\bar{\rho}^2$, so being negligible compared with the first term which is $\mathcal{O}(H^2)$, until $a \sim 1$. Then, equation (27) simplifies to

$$\frac{d}{dt} \left(\frac{\dot{B}^i}{a} \right) - \frac{B^{j,ji}}{a^3} = 0. \tag{28}$$

First, we show that once the perturbations enter the causal horizon, the longitudinal part of $B^i B^i/a^4$ in (21) undergoes damped oscillations, similar to those experienced by $\delta\Phi$. The longitudinal component of (28) in k -space reads

$$\frac{d}{dt} \left(\frac{\dot{B}_\parallel(k)}{a} \right) + \frac{k^2}{a^3} B_\parallel(k) = 0, \tag{29}$$

with the solution

$$B_\parallel(k) = a^{5/4} J_{5/2}(kd_c). \tag{30}$$

Hence B_\parallel oscillates inside $d_c = 2\Omega^{-1/2} H_0^{-1} a^{1/2}$ with the amplitude increasing linearly with a . As a consequence, the longitudinal term B_\parallel^2/a^4 that enters the right-hand side of (21) oscillates with an amplitude decreasing as a^{-2} .

Next we show that the transverse term $B_\perp^i B_\perp^i/a^4$ grows linearly. In the transverse components of (28) the last term is absent, so the transverse solution reads

$$B_\perp^i(a, \vec{x}) \simeq c^i(\vec{x}) \int_0^a \frac{da}{H} = \frac{2}{5} \frac{c^i(\vec{x})}{H_0 \Omega^{1/2}} a^{5/2}, \tag{31}$$

where Ω is the equivalent matter fraction at high redshift and $c^i(\vec{x})$ are arbitrary functions of \vec{x} . Thus,

$$\frac{B_\perp^i B_\perp^i}{a^4} = \frac{4}{25} \frac{c^i c^i}{H_0^2 \Omega} a \tag{32}$$

grows linearly with the scale factor. Owing to this equation we can arrange nonadiabatic perturbations such that

$$B_\perp^i B_\perp^i/a^4 - 3\delta = 0 \tag{33}$$

and hence $\delta p = 0$ with the assurance that this will hold independent of scale until $a \sim 1$. Here we denote the density contrast by $\delta = \delta\rho/\bar{\rho}$ for overdensities only. With vanishing c_s^2 , the spatial gradient term in (9) is absent, and the density contrast satisfies

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta = 0; \quad \delta > 0 \tag{34}$$

with the growing mode solution $\delta \propto a$. This is our main result: the growing mode overdensities here do not display the damped oscillations of the simple Chaplygin gas below d_s , but grow as dust. We remark that it matters little that this applies only for $\delta > 0$ since the Zel'dovich approximation implies that 92% ends up in overdense regions.

Clearly, there is an open question as to what inflation model can produce the initial conditions in the Kalb–Ramond field and the brane embedding to allow subsequent structure formation. We believe this issue is likely to be closely related to another: namely, how does the Chaplygin–Kalb–Ramond model fit into the braneworld picture? The estimate presented here can be taken as a starting point for investigating questions beyond linear theory, in the complete general-relativistic perturbation analysis including the electric-type field \mathcal{B}_{0i} . In

particular, one might hope that the acoustic horizon does resurrect at very small scales to provide the constant density cores seen in galaxies dominated by dark matter.

Ultimately, the model must be confronted with the large-scale structure and the CMB. In particular, the Kalb–Ramond field may leave a mark on the power spectrum and the CMB spectrum. To investigate this, one would need to do the full perturbation analysis, which would go beyond the scope of the present paper. For the simple Chaplygin gas where the standard model feels the metric $g_{\mu\nu}$, it has been shown [19] that a good fit to the data is obtained if a vanishing sound speed is imposed on the Zel’dovich fraction.

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